Paper 2: Pure Mathematics 2 Mark Scheme

Question	Scheme	Marks	AOs
1	Sets $\mathrm{f}(-2)=0 \Rightarrow 2\times(-2)^3-5\times(-2)^2+a\times-2+a=0$	M1	3.1a
	Solves linear equation $2a-a=-36 \Rightarrow a=$	dM1	1.1b
	$\Rightarrow a=-36$	A1	1.1b
		(3 marks)	

Notes:

M1: Selects a suitable method given that (x + 2) is a factor of f(x)
Accept either setting f(−2) = 0 or attempted division of f(x) by (x + 2)

dM1: Solves linear equation in a. Minimum requirement is that there are two terms in 'a' which must be collected to get $..a=..\Rightarrow a=$

A1: $a=-36$

Question	Scheme	Marks	AOs
2(a)	Identifies an error for student A: They use $\dfrac{\cos\theta}{\sin\theta}=\tan\theta$ It should be $\dfrac{\sin\theta}{\cos\theta}=\tan\theta$	B1	2.3
		(1)	
(b)	(i) Shows $\cos(-26.6°)\neq 2\sin(-26.6°)$, so cannot be a solution	B1	2.4
	(ii) Explains that the incorrect answer was introduced by squaring	B1	2.4
		(2)	
		(3 marks)	

Notes:

(a)

B1: Accept a response of the type 'They use $\dfrac{\cos\theta}{\sin\theta}=\tan\theta$. This is incorrect as $\dfrac{\sin\theta}{\cos\theta}=\tan\theta$'

It can be implied by a response such as 'They should get $\tan\theta=\dfrac{1}{2}$ not $\tan\theta=2$'

Accept also statements such as 'it should be $\cot\theta=2$'

(b)

B1: Accept a response where the candidate shows that $-26.6°$ is not a solution of $\cos\theta=2\sin\theta$. This can be shown by, for example, finding both $\cos(-26.6°)$ and $2\sin(-26.6°)$ and stating that they are not equal. An acceptable alternative is to state that $\cos(-26.6°)=+ve$ and $2\sin(-26.6°)=-ve$ and stating that they therefore cannot be equal.

B1: Explains that the incorrect answer was introduced by squaring Accept an example showing this. For example $x=5$ squared gives $x^2=25$ which has answers ± 5

Question	Scheme	Marks	AOs
3	Attempts the product and chain rule on $y = x(2x+1)^4$	M1	2.1
	$\frac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$	A1	1.1b
	Takes out a common factor $\frac{dy}{dx} = (2x+1)^3\{(2x+1)+8x\}$	M1	1.1b
	$\frac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n = 3, A = 10, B = 1$	A1	1.1b
			(4 marks)

Notes:

M1: Applies the product rule to reach $\frac{dy}{dx} = (2x+1)^4 + Bx(2x+1)^3$

A1: $\frac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$

M1: Takes out a common factor of $(2x+1)^3$

A1: The form of this answer is given. Look for $\frac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n = 3, A = 10, B = 1$

Question	Scheme	Marks	AOs
4 (a)	$\text{gf}(x) = 3\ln \text{e}^{x}$	M1	1.1b
	$= 3x, \; (x \in \quad)$	A1	1.1b
		(2)	
(b)	$\text{gf}(x) = \text{fg}(x) \Rightarrow 3x = x^3$	M1	1.1b
	$\Rightarrow x^3 - 3x = 0 \Rightarrow x =$	M1	1.1b
	$\Rightarrow x = (+)\sqrt{3}$ only as $\ln x$ is not defined at $x = 0$ and $-\sqrt{3}$	M1	2.2a
		(3)	
			(5 marks)

Notes:

(a)

M1: For applying the functions in the correct order

A1: The simplest form is required so it must be $3x$ and not left in the form $3\ln \text{e}^{x}$

An answer of $3x$ with no working would score both marks

(b)

M1: Allow the candidates to score this mark if they have $\text{e}^{3\ln x} =$ their $3x$

M1: For solving their cubic in x and obtaining at least one solution.

A1: For either stating that $x = \sqrt{3}$ **only** as $\ln x$ (or $3\ln x$) is not defined at $x = 0$ and $-\sqrt{3}$

or stating that $3x = x^3$ would have three answers, one positive one negative and one zero but $\ln x$ (or $3\ln x$) is not defined for $x \quad 0$ so therefore there is only one (real) answer.

Note: Student who mix up fg and gf can score full marks in part (b) as they have already been penalised in part (a)

Question	Scheme	Marks	AOs
5(a)	Substitutes $t=0.5$ into $m=25\mathrm{e}^{-0.05t} \Rightarrow m=25\mathrm{e}^{-0.05\times0.5}$	M1	3.4
	$\Rightarrow m=24.4\text{g}$	A1	1.1b
		(2)	
(b)	States or uses $\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{-0.05t}\right)=\pm C\,\mathrm{e}^{-0.05t}$	M1	2.1
	$\dfrac{\mathrm{d}m}{\mathrm{d}t}=-0.05\times25\mathrm{e}^{-0.05t}=-0.05m \Rightarrow k=-0.05$	A1	1.1b
		(2)	
			(4 marks)

Notes:

(a)

M1: Substitutes $t=0.5$ into $m=25\mathrm{e}^{-0.05t} \Rightarrow m=25\mathrm{e}^{-0.05\times0.5}$

A1: $m=24.4\text{g}$ An answer of $m=24.4\text{g}$ with no working would score both marks

(b)

M1: Applies the rule $\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{kx}\right)=k\,\mathrm{e}^{kx}$ in this context by stating or using $\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{-0.05t}\right)=\pm C\,\mathrm{e}^{-0.05t}$

A1: $\dfrac{\mathrm{d}m}{\mathrm{d}t}=-0.05\times25\mathrm{e}^{-0.05t}=-0.05m \Rightarrow k=-0.05$

Question	Scheme	Marks	AOs
6(i)	$x^2-6x+10=(x-3)^2+1$	M1	2.1
	Deduces "always true" as $(x-3)^2 \quad 0 \Rightarrow (x-3)^2+1 \quad 1$ and so is always positive	A1	2.2a
		(2)	
(ii)	For an explanation that it need not (always) be true This could be if $a<0$ then $ax>b \Rightarrow x<\frac{b}{a}$	M1	2.3
	States 'sometimes' and explains if $a>0$ then $ax>b \Rightarrow x>\frac{b}{a}$ if $a<0$ then $ax>b \Rightarrow x<\frac{b}{a}$	A1	2.4
		(2)	
(iii)	Difference $=(n+1)^2-n^2=2n+1$	M1	3.1a
	Deduces "Always true" as $2n+1$ = (even +1) = odd	A1	2.2a
		(2)	
			(6 marks)

Notes:

(i)

M1: Attempts to complete the square or any other valid reason. Allow for a graph of $y=x^2-6x+10$ or an attempt to find the minimum by differentiation

A1: States always true with a valid reason for their method

(ii)

M1: For an explanation that it need not be true (sometimes). This could be if $a<0$ then $ax>b \Rightarrow x<\frac{b}{a}$ or simply $-3x>6 \Rightarrow x<-2$

A1: Correct statement (sometimes true) and explanation

(iii)

M1: Sets up the proof algebraically.
For example by attempting $(n+1)^2-n^2=2n+1$ or $m^2-n^2=(m-n)(m+n)$ with $m=n+1$

A1: States always true with reason and proof
Accept a proof written in words. For example
If integers are consecutive, one is odd and one is even
When squared $\text{odd}\times\text{odd}=\text{odd}$ and $\text{even}\times\text{even}=\text{even}$
The difference between odd and even is always odd, hence always true
Score M1 for two of these lines and A1 for a good proof with all three lines or equivalent.

Question	Scheme	Marks	AOs
7(a)	$\sqrt{(4-x)} = 2\left(1-\frac{1}{4}x\right)^{\frac{1}{2}}$	M1	2.1
	$\left(1-\frac{1}{4}x\right)^{\frac{1}{2}} = 1+\frac{1}{2}\left(-\frac{1}{4}x\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(-\frac{1}{4}x\right)^{2}+\ldots$	M1	1.1b
	$\sqrt{(4-x)} = 2\left(1-\frac{1}{8}x-\frac{1}{128}x^{2}+..\right)$	A1	1.1b
	$\sqrt{(4-x)} = 2-\frac{1}{4}x-\frac{1}{64}x^{2}+\ldots$ and $k=-\frac{1}{64}$	A1	1.1b
		(4)	
(b)	The expansion is valid for $\lvert x\rvert<4$, so $x=1$ can be used	B1	2.4
		(1)	
			(5 marks)

Notes:

(a)

M1: Takes out a factor of 4 and writes $\sqrt{(4-x)} = 2(1\pm\ldots)^{\frac{1}{2}}$

M1: For an attempt at the binomial expansion with $n=\frac{1}{2}$

Eg. $(1+ax)^{\frac{1}{2}} = 1+\frac{1}{2}(ax)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(ax)^{2}+\ldots$

A1: Correct expression inside the bracket $1-\frac{1}{8}x-\frac{1}{128}x^{2}+$ which may be left unsimplified

A1: $\sqrt{(4-x)} = 2-\frac{1}{4}x-\frac{1}{64}x^{2}+\ldots$ and $k=-\frac{1}{64}$

(b)

B1: The expansion is valid for $\lvert x\rvert<4$, so $x=1$ can be used

Question	Scheme	Marks	AOs
8 (a)	Gradient $AB = -\frac{2}{5}$	B1	2.1
	y coordinate of A is 2	B1	2.1
	Uses perpendicular gradients $y = +\frac{5}{2}x + c$	M1	2.2a
	$\Rightarrow 2y - 5x = 4$ *	A1*	1.1b
		(4)	
(b)	Uses Pythagoras' theorem to find AB or AD Either $\sqrt{5^2+2^2}$ or $\sqrt{\left(\frac{4}{5}\right)^2+2^2}$	M1	3.1a
	Uses area $ABCD$ = $AD \times AB = \sqrt{29} \times \sqrt{\frac{116}{25}}$	M1	1.1b
	area $ABCD$ = 11.6	A1	1.1b
		(3)	
			(7 marks)

Notes:

(a) It is important that the student communicates each of these steps clearly

B1: States the gradient of AB is $-\frac{2}{5}$

B1: States that y coordinate of A = 2

M1: Uses the form $y = mx + c$ with $m =$ their adapted $-\frac{2}{5}$ and $c =$ their 2

Alternatively uses the form $y - y_1 = m(x - x_1)$ with $m =$ their adapted $-\frac{2}{5}$ and $(x_1, y_1) = (0, 2)$

A1*: Proceeds to given answer

(b)

M1: Finds the lengths of AB or AD using Pythagoras' Theorem. Look for $\sqrt{5^2+2^2}$ or $\sqrt{\left(\frac{4}{5}\right)^2+2^2}$

Alternatively finds the lengths BD and AO using coordinates. Look for $\left(5+\frac{4}{5}\right)$ and 2

M1: For a full method of finding the area of the rectangle $ABCD$. Allow for $AD \times AB$

Alternatively attempts area $ABCD$ = $2 \times \frac{1}{2} BD \times AO = 2 \times \frac{1}{2}$ '5.8'×'2'

A1: Area $ABCD$ = 11.6 or other exact equivalent such as $\frac{58}{5}$

Question	Scheme		Marks	AOs
9	$\int\left(3x^{0.5}+A\right)\mathrm{d}x = 2x^{1.5}+Ax\left(+c\right)$		M1 A1	3.1a 1.1b
	Uses limits and sets = $2A^2 \Rightarrow \left(2\times 8+4A\right)-\left(2\times 1+A\right)=2A^2$		M1	1.1b
	Sets up quadratic and attempts to solve	Sets up quadratic and attempts b^2-4ac	M1	1.1b
	$\Rightarrow A=-2, \frac{7}{2}$ **and** states that there are two roots	States $b^2-4ac=121>0$ and hence there are two roots	A1	2.4
				(5 marks)

Notes:

M1: Integrates the given function and achieves an answer of the form $kx^{1.5}+Ax\left(+c\right)$ where k is a non- zero constant

A1: Correct answer but may not be simplified

M1: Substitutes in limits and subtracts. This can only be scored if $\int A\,\mathrm{d}x = Ax$ and not $\frac{A^2}{2}$

M1: Sets up quadratic equation in A and either attempts to solve or attempts b^2-4ac

A1: Either $A=-2, \frac{7}{2}$ **and** states that there are two roots

Or states $b^2-4ac=121>0$ and hence there are two roots

Question	Scheme	Marks	AOs
10	Attempts $S_\infty = \frac{8}{7}\times S_6 \Rightarrow \frac{a}{1-r} = \frac{8}{7}\times\frac{a(1-r^6)}{1-r}$	M1	2.1
	$\Rightarrow 1 = \frac{8}{7}\times(1-r^6)$	M1	2.1
	$\Rightarrow r^6 = \frac{1}{8} \Rightarrow r = ..$	M1	1.1b
	$\Rightarrow r = \pm\frac{1}{\sqrt{2}}$ (so $k=2$)	A1	1.1b
			(4 marks)

Notes:

M1: Substitutes the correct formulae for S_∞ and S_6 into the given equation $S_\infty = \frac{8}{7}\times S_6$

M1: Proceeds to an equation just in r

M1: Solves using a correct method

A1: Proceeds to $r=\pm\frac{1}{\sqrt{2}}$ giving $k=2$

Question	Scheme	Marks	AOs
11 (a)	$\mathrm{f}(x) \quad 5$	B1	1.1b
		(1)	
(b)	Uses $-2(3-x)+5=\frac{1}{2}x+30$	M1	3.1a
	Attempts to solve by multiplying out bracket, collect terms etc $\frac{3}{2}x=31$	M1	1.1b
	$x=\frac{62}{3}$ only	A1	1.1b
		(3)	
(c)	Makes the connection that there must be two intersections. Implied by either end point $k>5$ or $k \quad 11$	M1	2.2a
	$\{k : k \in \quad , 5<k \quad 11\}$	A1	2.5
		(2)	
			(6 marks)

Notes:

(a)

B1: $\mathrm{f}(x) \quad 5$ Also allow $\mathrm{f}(x) \in [5, \infty)$

(b)

M1: Deduces that the solution to $\mathrm{f}(x)=\frac{1}{2}x+30$ can be found by solving

$$-2(3-x)+5=\frac{1}{2}x+30$$

M1: Correct method used to solve their equation. Multiplies out bracket/ collects like terms

A1: $x=\frac{62}{3}$ only. Do not allow 20.6

(c)

M1: Deduces that two distinct roots occurs when $y=k$ intersects $y=\mathrm{f}(x)$ in two places. This may be implied by the sight of either end point. Score for sight of either $k>5$ or $k \quad 11$

A1: Correct solution only $\{k : k \in \quad , 5<k \quad 11\}$

Question	Scheme	Marks	AOs
12(a)	Uses $\cos^2 x = 1 - \sin^2 x \Rightarrow 3\sin^2 x + \sin x + 8 = 9\left(1 - \sin^2 x\right)$	M1	3.1a
	$\Rightarrow 12\sin^2 x + \sin x - 1 = 0$	A1	1.1b
	$\Rightarrow \left(4\sin x - 1\right)\left(3\sin x + 1\right) = 0$	M1	1.1b
	$\Rightarrow \sin x = \frac{1}{4}, -\frac{1}{3}$	A1	1.1b
	Uses arcsin to obtain two correct values	M1	1.1b
	All four of $x = 14.48°, 165.52°, -19.47°, -160.53°$	A1	1.1b
		(6)	
(b)	Attempts $2\theta - 30° = -19.47°$	M1	3.1a
	$\Rightarrow \theta = 5.26°$	A1ft	1.1b
		(2)	
			(8 marks)

Notes:

(a)

M1: Substitutes $\cos^2 x = 1 - \sin^2 x$ into $3\sin^2 x + \sin x + 8 = 9\cos^2 x$ to create a quadratic equation in just $\sin x$

A1: $12\sin^2 x + \sin x - 1 = 0$ or exact equivalent

M1: Attempts to solve their quadratic equation in $\sin x$ by a suitable method. These could include factorisation, formula or completing the square.

A1: $\sin x = \frac{1}{4}, -\frac{1}{3}$

M1: Obtains two correct values for their $\sin x = k$

A1: All four of $x = 14.48°, 165.52°, -19.47°, -160.53°$

(b)

M1: For setting $2\theta - 30° = \text{their}\ '-19.47°'$

A1ft: $\theta = 5.26°$ but allow a follow through on their $'-19.47°'$

Question	Scheme	Marks	AOs
13(a)	$R=\sqrt{109}$	B1	1.1b
	$\tan\alpha=\dfrac{3}{10}$	M1	1.1b
	$\alpha=16.70°$ so $\sqrt{109}\cos(\theta+16.70°)$	A1	1.1b
		(3)	
(b)	(i) e.g $H=11-10\cos(80t)°+3\sin(80t)°$ or $H=11-\sqrt{109}\cos(80t+16.70)°$	B1	3.3
	(ii) $11+\sqrt{109}$ or 21.44 m	B1ft	3.4
		(2)	
(c)	Sets $80t+"16.70"=540$	M1	3.4
	$t=\dfrac{540-"16.70"}{80}=(6.54)$	M1	1.1b
	t = 6 mins 32 seconds	A1	1.1b
		(3)	
(d)	Increase the '80' in the formula For example use $H=11-10\cos(90t)°+3\sin(90t)°$		3.3
		(1)	
			(9 marks)

Notes:

(a)

B1: $R=\sqrt{109}$ Do not allow decimal equivalents

M1: Allow for $\tan\alpha=\pm\dfrac{3}{10}$

A1: $\alpha=16.70°$

(b)(i)

B1: see scheme

(b)(ii)

B1ft: their $11+$ their $\sqrt{109}$ Allow decimals here.

(c)

M1: Sets $80t+"16.70"=540$. Follow through on their 16.70

M1: Solves their $80t+"16.70"=540$ correctly to find t

A1: t = 6 mins 32 seconds

(d)

B1: States that to increase the speed of the wheel the 80's in the equation would need to be increased.

Question	Scheme	Marks	AOs
14(a)	Sets $500 = \pi r^2 h$	B1	2.1
	Substitute $h = \dfrac{500}{\pi r^2}$ into $S = 2\pi r^2 + 2\pi rh = 2\pi r^2 + 2\pi r \times \dfrac{500}{\pi r^2}$	M1	2.1
	Simplifies to reach given answer $S = 2\pi r^2 + \dfrac{1000}{r}$ *	A1*	1.1b
		(3)	
(b)	Differentiates S with both indices correct in $\dfrac{dS}{dr}$	M1	3.4
	$\dfrac{dS}{dr} = 4\pi r - \dfrac{1000}{r^2}$	A1	1.1b
	Sets $\dfrac{dS}{dr} = 0$ and proceeds to $r^3 = k$, k is a constant	M1	2.1
	Radius $= 4.30$ cm	A1	1.1b
	Substitutes their $r = 4.30$ into $h = \dfrac{500}{\pi r^2}$ $\Rightarrow$ Height $= 8.60$ cm	A1	1.1b
		(5)	
(c)	States a valid reason such as • The radius is too big for the size of our hands • If $r = 4.3$ cm and $h = 8.6$ cm the can is square in profile. All drinks cans are taller than they are wide • The radius is too big for us to drink from • They have different dimensions to other drinks cans and would be difficult to stack on shelves with other drinks cans	B1	3.2a
		(1)	
		9 marks	

Notes:

(a)

B1: Uses the correct volume formula with V =500. Accept $500 = \pi r^2 h$

M1: Substitutes $h = \dfrac{500}{\pi r^2}$ or $rh = \dfrac{500}{\pi r}$ into $S = 2\pi r^2 + 2\pi rh$ to get S as a function of r

A1*: $S = 2\pi r^2 + \dfrac{1000}{r}$ Note that this is a given answer.

(b)

M1: Differentiates the given S to reach $\dfrac{dS}{dr} = Ar \pm Br^{-2}$

A1: $\dfrac{dS}{dr} = 4\pi r - \dfrac{1000}{r^2}$ or exact equivalent

M1: Sets $\dfrac{dS}{dr} = 0$ and proceeds to $r^3 = k$, k is a constant

A1: R = awrt 4.30cm

A1: H = awrt 8.60 cm

(c)

B1: Any valid reason. See scheme for alternatives

Question	Scheme	Marks	AOs
15	$\dfrac{dy}{dx} = \dfrac{15}{2}x^{\frac{1}{2}} - 9$	M1 A1	3.1a 1.1b
	Substitutes $x = 4 \Rightarrow \dfrac{dy}{dx} = 6$	M1	2.1
	Uses (4, 15) and gradient $\Rightarrow y - 15 = 6(x - 4)$	M1	2.1
	Equation of l is $y = 6x - 9$	A1	1.1b
	Area $R = \displaystyle\int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11\right) - (6x - 9)\,dx$	M1	3.1a
	$= \left[2x^{\frac{5}{2}} - \dfrac{15}{2}x^2 + 20x(+c)\right]_0^4$	A1	1.1b
	Uses both limits of 4 and 0 $\left[2x^{\frac{5}{2}} - \dfrac{15}{2}x^2 + 20x\right]_0^4 = 2\times 4^{\frac{5}{2}} - \dfrac{15}{2}\times 4^2 + 20\times 4 - 0$	M1	2.1
	Area of R = 24 *	A1*	1.1b
	Correct notation with good explanations	A1	2.5
		(10)	
			(10 marks)

Question 15 continued

Notes:

M1: Differentiates $5x^{\frac{3}{2}} - 9x + 11$ to a form $Ax^{\frac{1}{2}} + B$

A1: $\frac{dy}{dx} = \frac{15}{2}x^{\frac{1}{2}} - 9$ but may not be simplified

M1: Substitutes $x = 4$ in their $\frac{dy}{dx}$ to find the gradient of the tangent

M1: Uses their gradient and the point (4, 15) to find the equation of the tangent

A1: Equation of l is $y = 6x - 9$

M1: Uses Area $R = \int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11\right) - (6x - 9)\,dx$ following through on their $y = 6x - 9$

Look for a form $Ax^{\frac{5}{2}} + Bx^2 + Cx$

A1: $= \left[2x^{\frac{5}{2}} - \frac{15}{2}x^2 + 20x(+c)\right]_0^4$ This must be correct but may not be simplified

M1: Substitutes in both limits and subtracts

A1*: Correct area for $R = 24$

A1: Uses correct notation and produces a well explained and accurate solution. Look for

- Correct notation used consistently and accurately for both differentiation and integration
- Correct explanations in producing the equation of l. See scheme.
- Correct explanation in finding the area of R. In way 2 a diagram may be used.

Alternative method for the area using area under curve and triangles. (Way 2)

M1: Area under curve = $\int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11\right) = \left[Ax^{\frac{5}{2}} + Bx^2 + Cx\right]_0^4$

A1: $= \left[2x^{\frac{5}{2}} - \frac{9}{2}x^2 + 11x\right]_0^4 = 36$

M1: This requires a full method with all triangles found using a correct method

Look for Area R = their $36 - \frac{1}{2} \times 15 \times \left(4 - \text{their } \frac{3}{2}\right) + \frac{1}{2} \times \text{their } 9 \times \text{their } \frac{3}{2}$

Question	Scheme	Marks	AOs
16(a)	Sets $\dfrac{1}{P(11-2P)} = \dfrac{A}{P} + \dfrac{B}{(11-2P)}$	B1	1.1a
	Substitutes either $P=0$ or $P=\dfrac{11}{2}$ into $1 = A(11-2P) + BP \Rightarrow A \text{ or } B$	M1	1.1b
	$\dfrac{1}{P(11-2P)} = \dfrac{1/11}{P} + \dfrac{2/11}{(11-2P)}$	A1	1.1b
		(3)	
(b)	Separates the variables $\displaystyle\int \frac{22}{P(11-2P)}\,\mathrm{d}P = \int 1\,\mathrm{d}t$	B1	3.1a
	Uses (a) and attempts to integrate $\displaystyle\int \frac{2}{P} + \frac{4}{(11-2P)}\,\mathrm{d}P = t + c$	M1	1.1b
	$2\ln P - 2\ln(11-2P) = t + c$	A1	1.1b
	Substitutes $t=0, P=1 \Rightarrow t=0, P=1 \Rightarrow c = (-2\ln 9)$	M1	3.1a
	Substitutes $P=2 \Rightarrow t = 2\ln 2 + 2\ln 9 - 2\ln 7$	M1	3.1a
	$\text{Time} = 1.89$ years	A1	3.2a
		(6)	
(c)	Uses ln laws $2\ln P - 2\ln(11-2P) = t - 2\ln 9$ $\Rightarrow \ln\left(\dfrac{9P}{11-2P}\right) = \dfrac{1}{2}t$	M1	2.1
	Makes 'P' the subject $\Rightarrow \left(\dfrac{9P}{11-2P}\right) = e^{\frac{1}{2}t}$ $\Rightarrow 9P = (11-2P)e^{\frac{1}{2}t}$ $\Rightarrow P = f\left(e^{\frac{1}{2}t}\right)$ or $\Rightarrow P = f\left(e^{-\frac{1}{2}t}\right)$	M1	2.1
	$\Rightarrow P = \dfrac{11}{2+9e^{-\frac{1}{2}t}} \Rightarrow A = 11, B = 2, C = 9$	A1	1.1b
		(3)	
		(12 marks)	

Question 16 continued

Notes:

(a)

B1: Sets $\frac{1}{P(11-2P)} = \frac{A}{P} + \frac{B}{(11-2P)}$

M1: Substitutes $P = 0$ or $P = \frac{11}{2}$ into $1 = A(11-2P) + BP \Rightarrow A \text{ or } B$

Alternatively compares terms to set up and solve two simultaneous equations in A and B

A1: $\frac{1}{P(11-2P)} = \frac{1/11}{P} + \frac{2/11}{(11-2P)}$ or equivalent $\frac{1}{P(11-2P)} = \frac{1}{11P} + \frac{2}{11(11-2P)}$

Note: The correct answer with no working scores all three marks.

(b)

B1: Separates the variables to reach $\int \frac{22}{P(11-2P)} \mathrm{d}P = \int 1 \mathrm{d}t$ or equivalent

M1: Uses part (a) and $\int \frac{A}{P} + \frac{B}{(11-2P)} \mathrm{d}P = A \ln P \pm C \ln(11-2P)$

A1: Integrates both sides to form a correct equation including a 'c' Eg $2\ln P - 2\ln(11-2P) = t + c$

M1: Substitutes t = 0 and P =1 to find c

M1: Substitutes P = 2 to find t. This is dependent upon having scored both previous M's

A1: Time = 1.89 years

(c)

M1: Uses correct log laws to move from $2\ln P - 2\ln(11-2P) = t + c$ to $\ln\left(\frac{P}{11-2P}\right) = \frac{1}{2}t + d$ for their numerical 'c'

M1: Uses a correct method to get P in terms of $\mathrm{e}^{\frac{1}{2}t}$

This can be achieved from $\ln\left(\frac{P}{11-2P}\right) = \frac{1}{2}t + d \Rightarrow \frac{P}{11-2P} = \mathrm{e}^{\frac{1}{2}t+d}$ followed by cross multiplication and collection of terms in P (See scheme)

Alternatively uses a correct method to get P in terms of $\mathrm{e}^{-\frac{1}{2}t}$ For example $\frac{P}{11-2P} = \mathrm{e}^{\frac{1}{2}t+d} \Rightarrow \frac{11-2P}{P} = \mathrm{e}^{-\left(\frac{1}{2}t+d\right)} \Rightarrow \frac{11}{P} - 2 = \mathrm{e}^{-\left(\frac{1}{2}t+d\right)} \Rightarrow \frac{11}{P} = 2 + \mathrm{e}^{-\left(\frac{1}{2}t+d\right)}$ followed by division

A1: Achieves the correct answer in the form required. $P = \frac{11}{2 + 9\mathrm{e}^{-\frac{1}{2}t}} \Rightarrow A = 11, B = 2, C = 9$ oe